Analysis of Seasonality in Sea Surface Salinity Balance Equation

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Background and Motivation

Dynamics of Sea Surface Salinity (SSS):

- Main determinants:
  freshwater forcing, horizontal advection, and vertical entrainment.
- Evolution of SSS and its determinants described by:

Upper Ocean Salinity Balance Equation

time derivative of SSS — horizontal advection

$$\frac{\partial S}{\partial t} = S_0 \frac{E-P}{h} - \vec{u}\cdot\nabla S - w\frac{\partial S}{\partial z}$$

surface freshwater forcing — vertical entrainment

- S_0: the constant 35
- E: evaporation
- P: precipitation
- h: mixed layer depth
- $\vec{u}$: the surface horizontal velocity
- w: the vertical velocity
- z: the vertical coordinate

- Change of freshwater forcing minus advection and vertical entrainment (FMAV) $\rightarrow$ change of SSS.
- Theoretically, when SSS has a sinusoidal annual cycle, lag between SSS and FMAV is expected to be **3 months**.

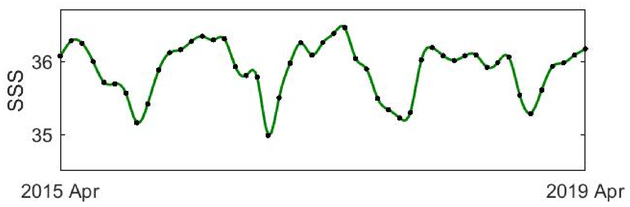
Motivation:

- Relationship between SSS and FMAV may be perturbed by the dynamics of the ocean.
- Estimate time lag (phase difference) between SSS and FMAV across time and space.
  $\Rightarrow$ Understand how changes in the ocean are impacted by forcing through the salinity balance equation.

Key Idea 1

Functional Data:

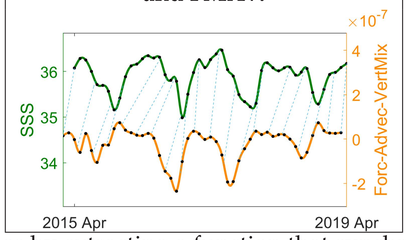
- Salinity evolves over time and space in a smooth fashion.
- In-situ measurement and remote sensing technology:
  observe at discrete time points only.
- It is natural to treat variables as functions rather than vectors of observations, known as functional data.



- Black markers: monthly values of SSS at a fixed location in the tropical North Atlantic Ocean.
- Green curve: underlying functional SSS trajectory obtained via smoothing monthly data.

Key Idea 2

Phase difference between SSS and FMAV:



- Estimate phase by extracting a function that synchronizes the two trajectories.
- Phase: represented by a function (varies across time)

$$\gamma\colon [0,1] \rightarrow [0,1],\ \gamma(0)=0,\ \gamma(1)=1,\ 0<\dot{\gamma}<\infty.$$

- Denoting SSS function by $s(t)$ and FMAV function by $f(t)$, observation model is

$$s(t) = (f\circ\gamma)(t) + e(t),$$

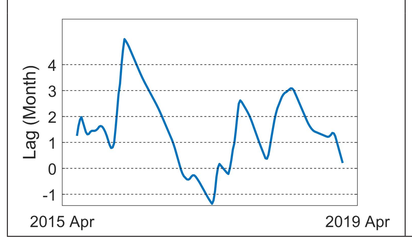
where $f\circ\gamma$ is domain warping of f via phase function γ and $e(t)$ is a discrepancy function.

- The estimator of the phase function aligning the two trajectories is given by:

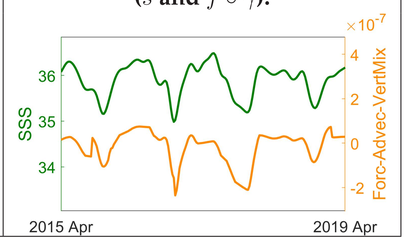
$$\hat{\gamma} = \underset{\gamma\in\Gamma}{\operatorname{arginf}} \|s-(f\circ\gamma)\|,$$

using a numerical algorithm.

Estimate phase function $\hat{\gamma}$:



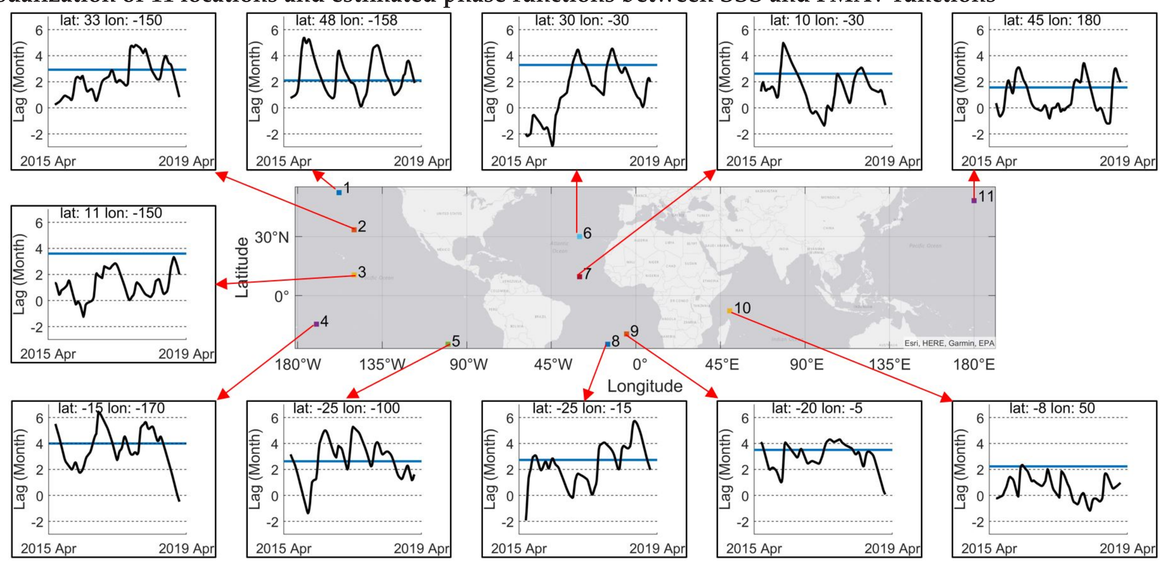
Phase synchronization (s and $f\circ\gamma$):



- **Traditional approach (Harmonic analysis)**:
  A constant phase estimate of 2.5 months.

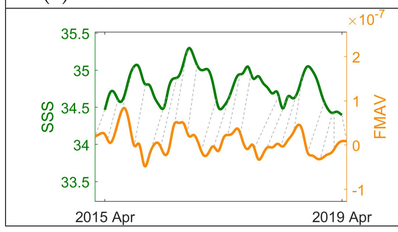
Modeling Results

Visualization of 11 locations and estimated phase functions between SSS and FMAV functions

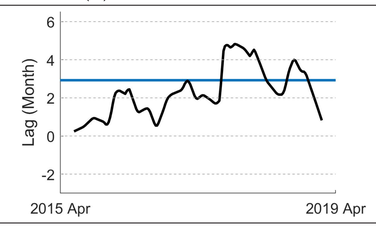


- Black function: Estimated phase function aligning FMAV to SSS.
- Blue line: Time lag between SSS and FMAV derived from *monthly* observations using harmonic analysis.

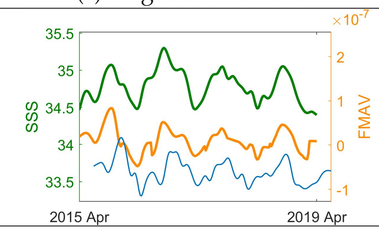
(a) SSS and FMAV of location 2



(b) Estimated Phase



(c) Aligned Functions



- Orange: FMAV aligned to SSS using phase function
- Blue: FMAV shifted by a constant phase difference